

Colorado Procedure 71-01

Standard Practice for

Determining Quality Level (Percent Within Tolerance Limits)

1. SCOPE

1.1 Use this procedure with Quality Assurance type specifications where Pay Factors or acceptance decisions are based on Quality Level (QL), defined as percent within specification (tolerance) limits. QL is a measure of quality of a lot or process.

1.2 QL represents the percentage of the population (lot or process) that falls above a single lower limit, below a single upper limit, or between the upper and lower limits of double-limit specifications.

1.3 For this procedure to be meaningful, select all samples by random or stratified random procedures. Perform all testing and measuring strictly in accordance with standard acceptable practices. When used for contractual purposes, perform all sampling and testing in accordance with the applicable specifications.

1.4 Manual, computer assisted, and mathematical procedures are described. Where contractual pay factors are based on QL, use only the computer assisted procedure.

2. SUMMARY OF METHOD

2.1 The method involves calculating statistical parameters from three or more representative measurements, test results, or values for each specified element in a lot or sample. The arithmetic average (mean) value of the sample is calculated. As a measure of variability, the sample Standard Deviation is calculated. Using these results, the distance from the sample mean to each limit is divided by the standard deviation, which yields the Quality Index.

2.2 The incomplete beta function ratio, using sample sizes and quality indices as variables, is used in the computer version to calculate areas under the beta distribution. With variables typical for QL determinations, the beta

distribution (Figure 71-1) is similar to the normal distribution (Figure 71-2).

2.3 The total area under the beta distribution outside the specification limits is the fraction defective, which is then multiplied by 100 to yield the percent defective; this subtracted from 100 gives the percent within limits.

2.4 Table 71-1 contains values for percent within limits as related to sample sizes and quality indices. The table was developed from mathematical calculations and is used in the manual method to estimate QL.

3. MANUAL PROCEDURE

3.1 Determine the arithmetic mean and standard deviation for the several test results from the lot for each element being evaluated. Compute these as shown in Equations 3.1 and 3.2.

$$\bar{X} = \frac{\sum X}{n} \quad \text{Equation 3.1}$$

$$s = \sqrt{\frac{\sum (X - \bar{X})^2}{n - 1}} \quad \text{Equation 3.2}$$

Where:

- \bar{X} = Sample mean,
- \sum = Summation of,
- X = Individual test value to X_n ,
- n = Total number of test values,
- s = Sample standard deviation.

3.2 Compute the upper quality index (Q_u) per Equation 3.3.

$$Q_u = \frac{T_u - \bar{X}}{s} \quad \text{Equation 3.3}$$

Where:

Q_u = Upper quality index,
 T_u = Upper specification limits.

3.2.1 Determine P_u (percent within the upper specification limit which corresponds to a given Q_u) from Table 71-1. If desired, P_u may be interpolated to the nearest 0.1. Where T_u is not specified, P_u will be 100.

3.3 Compute the lower quality index (Q_L) per Equation 3.4.

$$Q_L = \frac{\bar{X} - T_L}{s} \quad \text{Equation 3.4}$$

Where:

Q_L = Lower quality index,
 T_L = Lower specification limits.

3.3.1 Determine P_L (percent within the lower specification limit which corresponds to a given Q_L) from Table 71-1. If desired, P_L may be interpolated to the nearest 0.1. Where T_L is not specified, P_L will be 100.

3.4 Compute QL (the total percent within specification limits) per Equation 3.5.

$$QL = (P_u + P_L) - 100 \quad \text{Equation 3.5}$$

3.5 The manual method for determining QL essentially conforms to the applicable portions of AASHTO Standard Recommended Practice R 9, Acceptance Sampling Plans for Highway Construction.

3.6 A sample calculation is provided at the end of this procedure demonstrating the

calculation of Quality Level and Pay Factors using this manual procedure.

4. COMPUTER ASSISTED PROCEDURE

4.1 The calculations for determining Quality Level may be performed by using the latest versions of the Departments quality level programs.

4.2 In the quality level programs, the areas under the beta distribution are calculated from the incomplete beta function ratio by assigning the variables used in Equations 3.1 through 3.4. The procedure is as described in *Numerical Recipes in C₁, Chapter 6*. A detailed discussion of the theories involved is provided by Willenbrock and Kopac in *TRR 691, Process Control in the Construction Industry₂*.

4.3 All numbers from the calculations are carried to significant figures and round according to AASHTO Standard Recommended Practice R 11, using the Rounding Method.

4.4 Where contractual pay factors are based on the QL use the CDOT Application for Reporting (CAR) reports for obtaining the incentive/disincentive for the item. The quantity shown on the CAR report shall match the total quantity paid on the estimate. Differences in quantities shall be explained on the CDOT Form 473 page 2, Explanation of Exceptions.

MATHEMATICAL PROCEDURE - Adapted from *Resolution of beta-distribution equations for quality level analysis...*

5.1 In order to evaluate the necessary quality parameters, the integral must be evaluated.

$$I_n = \frac{1}{B\left(\frac{n}{2} - 1, \frac{n}{2} - 1\right)} \int_0^g t^{\frac{n}{2} - 2} (1 - t)^{\frac{n}{2} - 2} dt \quad \text{Equation 5.1}$$

In Equation 5.1 $B(n/2-1, n/2-1)$ is generally referred to as the complete beta-function (or just the beta-function) with parameters $n/2-1, n/2-1$, and the integral is the incomplete beta-function. Together they form the beta distribution from a random variable. The beta function is defined by:

$$B\left(\frac{n}{2} - 1, \frac{n}{2} - 1\right) = \int_0^1 t^{\frac{n}{2} - 2} (1 - t)^{\frac{n}{2} - 2} dt, \quad \text{Equation 5.2}$$

and the upper limit in Equation 5.1 is given by:

$$g = \frac{1}{2} - \frac{Q\sqrt{n}}{2(n - 1)} \quad \text{Equation 5.3}$$

where Q is the quality index defined in Equations 3.3 and 3.4 and n is the sample size.

5.2 For small sample sizes no numerical integration is necessary as the integral may be economically evaluated in close form. In particular we have:

$$I_3 = \frac{1}{2} + \frac{1}{p} \sin^{-1}(2g - 1) \quad \text{Equation 5.4}$$

$$I_4 = g \quad \text{Equation 5.5}$$

$$I_5 = \frac{1}{2} + \frac{1}{p} \sin^{-1}(2g - 1) + \frac{2}{p} \sqrt{g - g^2} (2g - 1) \quad \text{Equation 5.6}$$

$$I_6 = 3g^2 - 2g^3 \quad \text{Equation 5.7}$$

$$I_7 = \frac{1}{2} + \frac{1}{p} \sin^{-1}(2g - 1) - \frac{2}{3p} \sqrt{g - g^2} (2g - 1)(8g^2 - 8g - 3) \quad \text{Equation 5.8}$$

$$I_8 = 10g^3 - 15g^4 + 6g^5 \quad \text{Equation 5.9}$$

These expressions are small enough to be used with some hand calculators. As the value of n increases the calculations become more complex. With the availability of personal computers, we include the equation for information and recommend the use of personal computers.

TABLE 71-1

| Upper Quality Index Q_u or Lower Quality Index Q_L | | | | | | | | | | | | | | | | |
|--|------|------|------|------|------|------|------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------|--------------------|--|
| P_u or P_L % | | | | | | | | n=10 to n=11 | n=12 to n=14 | n=15 to n=18 | n=19 to n=25 | n=26 to n=37 | n=38 to n=69 | n=70 to n=200 | n=201 to n=x | |
| | n=3 | n=4 | n=5 | n=6 | n=7 | n=8 | n=9 | | | | | | | | | |
| 100 | 1.16 | 1.50 | 1.79 | 2.03 | 2.23 | 2.39 | 2.53 | 2.65 | 2.83 | 3.03 | 3.20 | 3.38 | 3.54 | 3.70 | 3.83 | |
| 99 | | 1.47 | 1.67 | 1.80 | 1.89 | 1.95 | 2.00 | 2.04 | 2.09 | 2.14 | 2.18 | 2.22 | 2.26 | 2.29 | 2.31 | |
| 98 | 1.15 | 1.44 | 1.60 | 1.70 | 1.76 | 1.81 | 1.84 | 1.86 | 1.91 | 1.93 | 1.96 | 1.99 | 2.01 | 2.03 | 2.05 | |
| 97 | | 1.41 | 1.54 | 1.62 | 1.67 | 1.70 | 1.72 | 1.74 | 1.77 | 1.79 | 1.81 | 1.83 | 1.85 | 1.86 | 1.87 | |
| 96 | 1.14 | 1.38 | 1.49 | 1.55 | 1.59 | 1.61 | 1.63 | 1.65 | 1.67 | 1.68 | 1.70 | 1.71 | 1.73 | 1.74 | 1.75 | |
| 95 | | 1.35 | 1.44 | 1.49 | 1.52 | 1.54 | 1.55 | 1.56 | 1.58 | 1.59 | 1.61 | 1.62 | 1.63 | 1.63 | 1.64 | |
| 94 | 1.13 | 1.32 | 1.39 | 1.43 | 1.46 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.52 | 1.53 | 1.54 | 1.55 | 1.55 | |
| 93 | | 1.29 | 1.35 | 1.38 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.44 | 1.45 | 1.46 | 1.46 | 1.47 | 1.47 | |
| 92 | 1.12 | 1.26 | 1.31 | 1.33 | 1.35 | 1.36 | 1.36 | 1.36 | 1.37 | 1.37 | 1.39 | 1.39 | 1.40 | 1.40 | 1.40 | |
| 91 | 1.11 | 1.23 | 1.27 | 1.29 | 1.30 | 1.30 | 1.31 | 1.31 | 1.32 | 1.32 | 1.33 | 1.33 | 1.33 | 1.34 | 1.34 | |
| 90 | 1.10 | 1.20 | 1.23 | 1.24 | 1.25 | 1.25 | 1.26 | 1.26 | 1.26 | 1.27 | 1.27 | 1.27 | 1.28 | 1.28 | 1.28 | |
| 89 | 1.09 | 1.17 | 1.19 | 1.20 | 1.20 | 1.21 | 1.21 | 1.21 | 1.21 | 1.22 | 1.22 | 1.22 | 1.22 | 1.22 | 1.23 | |
| 88 | 1.07 | 1.14 | 1.15 | 1.16 | 1.16 | 1.16 | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 | 1.17 | |
| 87 | 1.06 | 1.11 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.13 | 1.13 | |
| 86 | 1.04 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | |
| 85 | 1.03 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 | |
| 84 | 1.01 | 1.02 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.99 | 0.99 | 0.99 | |
| 83 | 1.00 | 0.99 | 0.98 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | |
| 82 | 0.97 | 0.96 | 0.95 | 0.94 | 0.93 | 0.93 | 0.93 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | |
| 81 | 0.96 | 0.93 | 0.91 | 0.90 | 0.90 | 0.89 | 0.89 | 0.89 | 0.89 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | |
| 80 | 0.93 | 0.90 | 0.88 | 0.87 | 0.86 | 0.86 | 0.86 | 0.85 | 0.85 | 0.85 | 0.85 | 0.84 | 0.84 | 0.84 | 0.84 | |
| 79 | 0.91 | 0.87 | 0.85 | 0.84 | 0.83 | 0.82 | 0.82 | 0.82 | 0.82 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 | 0.81 | |
| 78 | 0.89 | 0.84 | 0.82 | 0.80 | 0.80 | 0.79 | 0.79 | 0.79 | 0.78 | 0.78 | 0.78 | 0.78 | 0.77 | 0.77 | 0.77 | |
| 77 | 0.87 | 0.81 | 0.78 | 0.77 | 0.76 | 0.76 | 0.76 | 0.75 | 0.75 | 0.75 | 0.75 | 0.74 | 0.74 | 0.74 | 0.74 | |
| 76 | 0.84 | 0.78 | 0.75 | 0.74 | 0.73 | 0.73 | 0.72 | 0.72 | 0.72 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | 0.71 | |
| 75 | 0.82 | 0.75 | 0.72 | 0.71 | 0.70 | 0.70 | 0.69 | 0.69 | 0.69 | 0.68 | 0.68 | 0.68 | 0.68 | 0.68 | 0.67 | |
| 74 | 0.79 | 0.72 | 0.69 | 0.68 | 0.67 | 0.66 | 0.66 | 0.66 | 0.66 | 0.65 | 0.65 | 0.65 | 0.65 | 0.64 | 0.64 | |
| 73 | 0.76 | 0.69 | 0.66 | 0.65 | 0.64 | 0.63 | 0.63 | 0.63 | 0.62 | 0.62 | 0.62 | 0.62 | 0.62 | 0.61 | 0.61 | |
| 72 | 0.74 | 0.66 | 0.63 | 0.62 | 0.61 | 0.60 | 0.60 | 0.60 | 0.59 | 0.59 | 0.59 | 0.59 | 0.59 | 0.58 | 0.58 | |
| 71 | 0.71 | 0.63 | 0.60 | 0.59 | 0.58 | 0.57 | 0.57 | 0.57 | 0.57 | 0.56 | 0.56 | 0.56 | 0.56 | 0.55 | 0.55 | |
| 70 | 0.68 | 0.60 | 0.57 | 0.56 | 0.55 | 0.55 | 0.54 | 0.54 | 0.54 | 0.53 | 0.53 | 0.53 | 0.53 | 0.53 | 0.52 | |
| 69 | 0.65 | 0.57 | 0.54 | 0.53 | 0.52 | 0.52 | 0.51 | 0.51 | 0.51 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | 0.50 | |
| 68 | 0.62 | 0.54 | 0.51 | 0.50 | 0.49 | 0.49 | 0.48 | 0.48 | 0.48 | 0.48 | 0.47 | 0.47 | 0.47 | 0.47 | 0.47 | |
| 67 | 0.59 | 0.51 | 0.47 | 0.47 | 0.46 | 0.46 | 0.46 | 0.45 | 0.45 | 0.45 | 0.45 | 0.44 | 0.44 | 0.44 | 0.44 | |
| 66 | 0.56 | 0.48 | 0.45 | 0.44 | 0.44 | 0.43 | 0.43 | 0.43 | 0.42 | 0.42 | 0.42 | 0.42 | 0.41 | 0.41 | 0.41 | |
| 65 | 0.52 | 0.45 | 0.43 | 0.41 | 0.41 | 0.40 | 0.40 | 0.40 | 0.40 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | 0.39 | |
| 64 | 0.49 | 0.42 | 0.40 | 0.39 | 0.38 | 0.38 | 0.37 | 0.37 | 0.37 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | 0.36 | |
| 63 | 0.46 | 0.39 | 0.37 | 0.36 | 0.35 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.33 | 0.33 | 0.33 | |
| 62 | 0.43 | 0.36 | 0.34 | 0.33 | 0.32 | 0.32 | 0.32 | 0.32 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | |
| 61 | 0.39 | 0.33 | 0.31 | 0.30 | 0.30 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.28 | 0.28 | 0.28 | 0.28 | 0.28 | |
| 60 | 0.36 | 0.30 | 0.28 | 0.27 | 0.27 | 0.27 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.26 | 0.25 | 0.25 | |
| 59 | 0.32 | 0.27 | 0.25 | 0.25 | 0.24 | 0.24 | 0.24 | 0.24 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | |
| 58 | 0.29 | 0.24 | 0.23 | 0.22 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | |
| 57 | 0.25 | 0.21 | 0.20 | 0.19 | 0.19 | 0.19 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | 0.18 | |
| 56 | 0.22 | 0.18 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | |
| 55 | 0.18 | 0.15 | 0.14 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | |
| 54 | 0.14 | 0.12 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | |
| 53 | 0.11 | 0.09 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | |
| 52 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | |
| 51 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | |
| 50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | |

NOTE: When Q_u or Q_L falls between table values, estimate P_u or P_L to the closest 0.10.

TABLE 71-1

| Upper Quality Index Q_u or Lower Quality Index Q_L | | | | | | | | | | | | | | | |
|--|-------|-------|-------|-------|-------|-------|-------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------|--------------------|
| P_u or P_L % | | | | | | | | n=10 to n=11 | n=12 to n=14 | n=15 to n=18 | n=19 to n=25 | n=26 to n=37 | n=38 to n=69 | n=70 to n=200 | n=201 to n=x |
| | 50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 49 | -0.04 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.03 | -0.02 |
| 48 | -0.07 | -0.06 | -0.06 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 | -0.05 |
| 47 | -0.11 | -0.09 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 | -0.08 |
| 46 | -0.14 | -0.12 | -0.11 | -0.11 | -0.11 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 | -0.10 |
| 45 | -0.18 | -0.15 | -0.14 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 | -0.13 |
| 44 | -0.22 | -0.18 | -0.16 | -0.16 | -0.16 | -0.16 | -0.16 | -0.16 | -0.16 | -0.15 | -0.15 | -0.15 | -0.15 | -0.15 | -0.15 |
| 43 | -0.25 | -0.21 | -0.20 | -0.19 | -0.19 | -0.19 | -0.18 | -0.18 | -0.18 | -0.18 | -0.18 | -0.18 | -0.18 | -0.18 | -0.18 |
| 42 | -0.29 | -0.24 | -0.23 | -0.22 | -0.21 | -0.21 | -0.21 | -0.21 | -0.21 | -0.21 | -0.20 | -0.20 | -0.20 | -0.20 | -0.20 |
| 41 | -0.32 | -0.27 | -0.25 | -0.25 | -0.24 | -0.24 | -0.24 | -0.24 | -0.23 | -0.23 | -0.23 | -0.23 | -0.23 | -0.23 | -0.23 |
| 40 | -0.36 | -0.30 | -0.28 | -0.27 | -0.27 | -0.27 | -0.26 | -0.26 | -0.26 | -0.26 | -0.26 | -0.26 | -0.26 | -0.25 | -0.25 |
| 39 | -0.39 | -0.33 | -0.31 | -0.30 | -0.30 | -0.29 | -0.29 | -0.29 | -0.29 | -0.29 | -0.28 | -0.28 | -0.28 | -0.28 | -0.28 |
| 38 | -0.43 | -0.36 | -0.34 | -0.33 | -0.32 | -0.32 | -0.32 | -0.32 | -0.31 | -0.31 | -0.31 | -0.31 | -0.31 | -0.31 | -0.31 |
| 37 | -0.46 | -0.39 | -0.37 | -0.36 | -0.35 | -0.35 | -0.35 | -0.34 | -0.34 | -0.34 | -0.34 | -0.34 | -0.33 | -0.33 | -0.33 |
| 36 | -0.49 | -0.42 | -0.40 | -0.39 | -0.38 | -0.38 | -0.37 | -0.37 | -0.37 | -0.36 | -0.36 | -0.36 | -0.36 | -0.36 | -0.36 |
| 35 | -0.52 | -0.45 | -0.43 | -0.41 | -0.41 | -0.40 | -0.40 | -0.40 | -0.40 | -0.39 | -0.39 | -0.39 | -0.39 | -0.39 | -0.39 |
| 34 | -0.56 | -0.48 | -0.45 | -0.44 | -0.44 | -0.43 | -0.43 | -0.43 | -0.42 | -0.42 | -0.42 | -0.42 | -0.41 | -0.41 | -0.41 |
| 33 | -0.59 | -0.51 | -0.47 | -0.47 | -0.46 | -0.46 | -0.46 | -0.45 | -0.45 | -0.45 | -0.45 | -0.44 | -0.44 | -0.44 | -0.44 |
| 32 | -0.62 | -0.54 | -0.51 | -0.50 | -0.49 | -0.49 | -0.48 | -0.48 | -0.48 | -0.48 | -0.47 | -0.47 | -0.47 | -0.47 | -0.47 |
| 31 | -0.65 | -0.57 | -0.54 | -0.53 | -0.52 | -0.52 | -0.51 | -0.51 | -0.51 | -0.50 | -0.50 | -0.50 | -0.50 | -0.50 | -0.50 |
| 30 | -0.68 | -0.60 | -0.57 | -0.56 | -0.55 | -0.55 | -0.54 | -0.54 | -0.54 | -0.53 | -0.53 | -0.53 | -0.53 | -0.53 | -0.52 |
| 29 | -0.71 | -0.63 | -0.60 | -0.59 | -0.58 | -0.57 | -0.57 | -0.57 | -0.57 | -0.56 | -0.56 | -0.56 | -0.56 | -0.55 | -0.55 |
| 28 | -0.74 | -0.66 | -0.63 | -0.62 | -0.61 | -0.60 | -0.60 | -0.60 | -0.59 | -0.59 | -0.59 | -0.59 | -0.59 | -0.58 | -0.58 |
| 27 | -0.76 | -0.69 | -0.66 | -0.65 | -0.64 | -0.63 | -0.63 | -0.63 | -0.62 | -0.62 | -0.62 | -0.62 | -0.62 | -0.61 | -0.61 |
| 26 | -0.79 | -0.72 | -0.69 | -0.68 | -0.67 | -0.66 | -0.66 | -0.66 | -0.66 | -0.65 | -0.65 | -0.65 | -0.65 | -0.64 | -0.64 |
| 25 | -0.82 | -0.75 | -0.72 | -0.71 | -0.70 | -0.70 | -0.69 | -0.69 | -0.69 | -0.68 | -0.68 | -0.68 | -0.68 | -0.68 | -0.67 |
| 24 | -0.84 | -0.78 | -0.75 | -0.74 | -0.73 | -0.73 | -0.72 | -0.72 | -0.72 | -0.71 | -0.71 | -0.71 | -0.71 | -0.71 | -0.71 |
| 23 | -0.87 | -0.81 | -0.78 | -0.77 | -0.76 | -0.76 | -0.76 | -0.75 | -0.75 | -0.75 | -0.75 | -0.74 | -0.74 | -0.74 | -0.74 |
| 22 | -0.89 | -0.84 | -0.82 | -0.80 | -0.80 | -0.79 | -0.79 | -0.79 | -0.78 | -0.78 | -0.78 | -0.78 | -0.77 | -0.77 | -0.77 |
| 21 | -0.91 | -0.87 | -0.85 | -0.84 | -0.83 | -0.82 | -0.82 | -0.82 | -0.82 | -0.81 | -0.81 | -0.81 | -0.81 | -0.81 | -0.81 |
| 20 | -0.93 | -0.90 | -0.88 | -0.87 | -0.86 | -0.86 | -0.86 | -0.85 | -0.85 | -0.85 | -0.85 | -0.84 | -0.84 | -0.84 | -0.84 |
| 19 | -0.96 | -0.93 | -0.91 | -0.90 | -0.90 | -0.89 | -0.89 | -0.89 | -0.89 | -0.88 | -0.88 | -0.88 | -0.88 | -0.88 | -0.88 |
| 18 | -0.97 | -0.96 | -0.95 | -0.94 | -0.93 | -0.93 | -0.93 | -0.92 | -0.92 | -0.92 | -0.92 | -0.92 | -0.92 | -0.92 | -0.92 |
| 17 | -1.00 | -0.99 | -0.98 | -0.97 | -0.96 | -0.96 | -0.96 | -0.96 | -0.96 | -0.96 | -0.96 | -0.96 | -0.95 | -0.95 | -0.95 |
| 16 | -1.01 | -1.02 | -1.01 | -1.01 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -1.00 | -0.99 | -0.99 | -0.99 |
| 15 | -1.03 | -1.05 | -1.05 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 | -1.04 |
| 14 | -1.04 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 | -1.08 |
| 13 | -1.06 | -1.11 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.12 | -1.13 | -1.13 |
| 12 | -1.07 | -1.14 | -1.15 | -1.16 | -1.16 | -1.16 | -1.16 | -1.17 | -1.17 | -1.17 | -1.17 | -1.17 | -1.17 | -1.17 | -1.17 |
| 11 | -1.09 | -1.17 | -1.19 | -1.20 | -1.20 | -1.21 | -1.21 | -1.21 | -1.21 | -1.22 | -1.22 | -1.22 | -1.22 | -1.22 | -1.23 |
| 10 | -1.10 | -1.20 | -1.23 | -1.24 | -1.25 | -1.25 | -1.26 | -1.26 | -1.26 | -1.27 | -1.27 | -1.27 | -1.28 | -1.28 | -1.28 |
| 9 | -1.11 | -1.23 | -1.27 | -1.29 | -1.30 | -1.30 | -1.31 | -1.31 | -1.32 | -1.32 | -1.33 | -1.33 | -1.33 | -1.34 | -1.34 |
| 8 | -1.12 | -1.26 | -1.31 | -1.33 | -1.35 | -1.36 | -1.36 | -1.36 | -1.37 | -1.37 | -1.39 | -1.39 | -1.40 | -1.40 | -1.40 |
| 7 | -1.13 | -1.29 | -1.35 | -1.38 | -1.40 | -1.41 | -1.42 | -1.43 | -1.44 | -1.44 | -1.45 | -1.46 | -1.46 | -1.47 | -1.47 |
| 6 | -1.13 | -1.32 | -1.39 | -1.43 | -1.46 | -1.47 | -1.48 | -1.49 | -1.50 | -1.51 | -1.52 | -1.53 | -1.54 | -1.55 | -1.55 |
| 5 | -1.14 | -1.35 | -1.44 | -1.49 | -1.52 | -1.54 | -1.55 | -1.56 | -1.58 | -1.59 | -1.61 | -1.62 | -1.63 | -1.63 | -1.64 |
| 4 | -1.14 | -1.38 | -1.49 | -1.55 | -1.59 | -1.61 | -1.63 | -1.65 | -1.67 | -1.68 | -1.70 | -1.71 | -1.73 | -1.74 | -1.75 |
| 3 | -1.15 | -1.41 | -1.54 | -1.62 | -1.67 | -1.70 | -1.72 | -1.74 | -1.77 | -1.79 | -1.81 | -1.83 | -1.85 | -1.86 | -1.87 |
| 2 | -1.15 | -1.44 | -1.60 | -1.70 | -1.76 | -1.81 | -1.84 | -1.86 | -1.91 | -1.93 | -1.96 | -1.99 | -2.01 | -2.03 | -2.05 |
| 1 | -1.15 | -1.47 | -1.67 | -1.80 | -1.89 | -1.95 | -2.00 | -2.04 | -2.09 | -2.14 | -2.18 | -2.22 | -2.26 | -2.29 | -2.31 |
| 0 | -1.16 | -1.50 | -1.79 | -2.03 | -2.23 | -2.39 | -2.53 | -2.65 | -2.83 | -3.03 | -3.20 | -3.38 | -3.54 | -3.70 | -3.83 |

NOTE: When Q_u or Q_L falls between table values, estimate P_u or P_L to the closest 0.10.

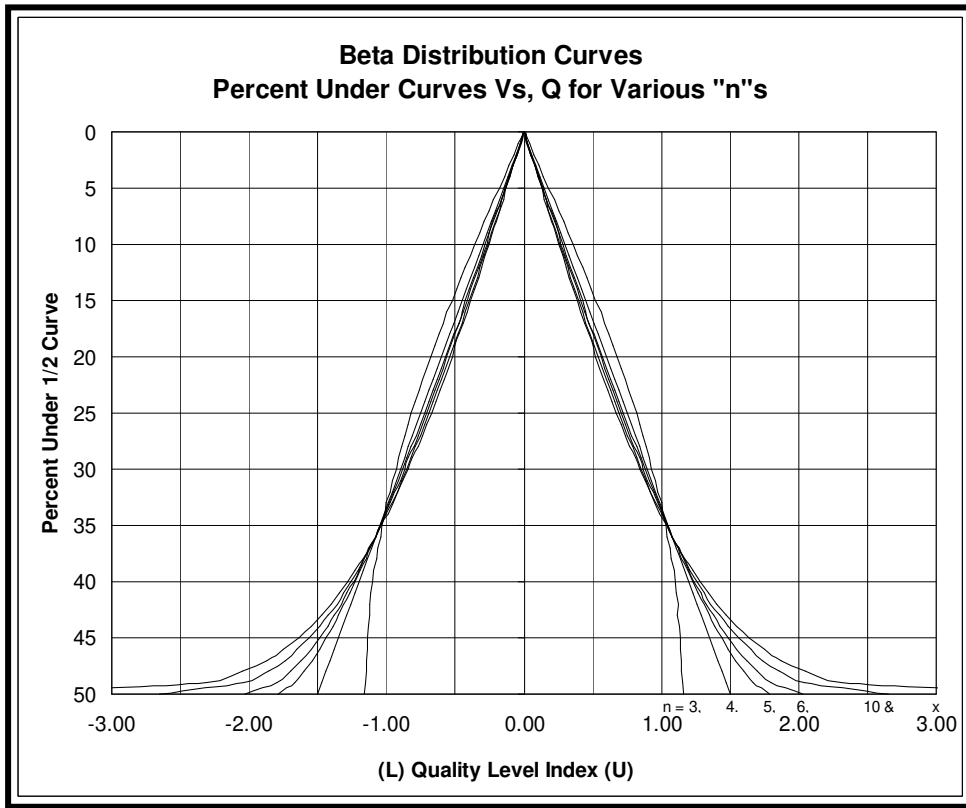


Figure 71-1

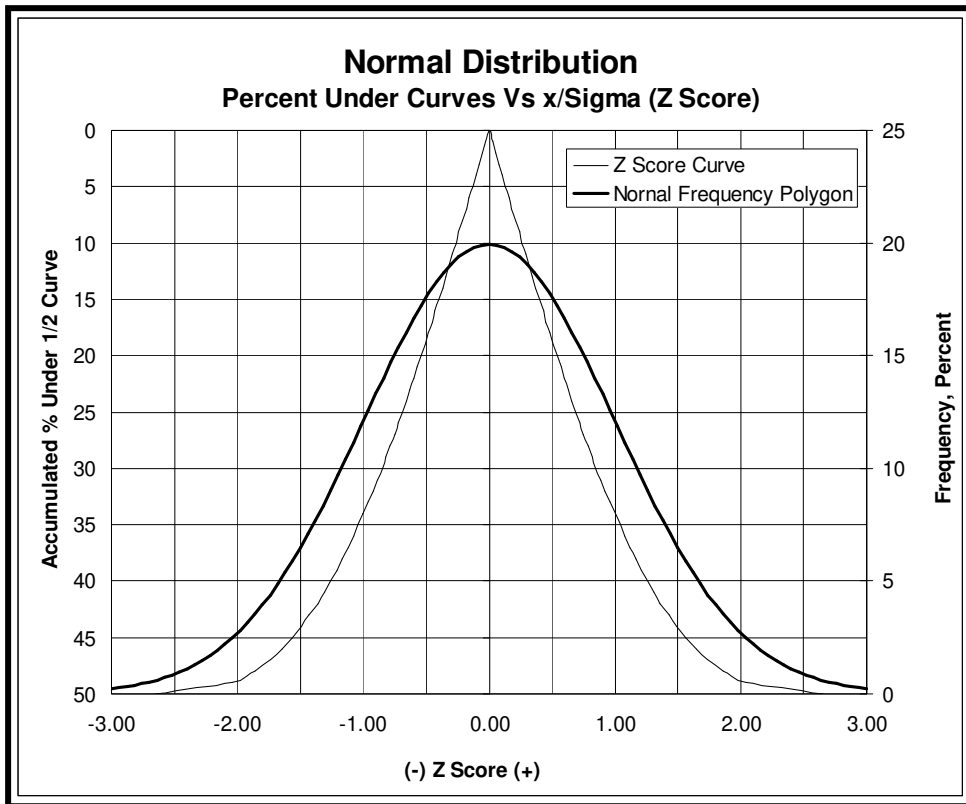


Figure 71-2

Footnotes:

1. Numerical Recipes in C, the Art of Scientific Computing; by W. H. Press, B.P. Flannery, S. A. Teukolsky and W.T. Vetterling. Cambridge University Press, The Pitt Bldg, Trumpington Street, CB2 1RP, 40 West 20th St., New York, NY 10011. Copyright 1988.
2. Development of a Highway Acceptance Plan, by Jack H. Willenbrock, Pennsylvania State University and Peter A. Kopac, Federal Highway Administration. TRR 691, Process Control in the Construction Industry, National Academy of Sciences, Washington, D.C. 1978.
3. Resolution of Beta-Distribution Formulas for Quality Level Analysis, a report to the Colorado Department of Transportation from the Colorado Workshop on Mathematical Problems in Industry, prepared by F. Jay Bourland, Department of Mathematics, Colorado State University and Alistair Fitt, Department of Mathematics, University of Southampton.

Determining Quality Level and Pay Factor

Sample Calculation for Close Approximation - Manual Procedure

To Solve for Quality Level:

This example will demonstrate the manual calculation of the Quality Level for asphalt density testing. With a nuclear gauge, the following test results were obtained: 92.5, 93.4, 94.8, 95.2, and 96.4. Using density specification of 92 to 96 percent compaction.

Use Equations 3.1 and 3.2, to solve for the arithmetic mean and the standard deviation.

Arithmetic mean: 94.46
Standard Deviation: 1.532

With those known, use Equations 3.3 and 3.4 to solve for the upper quality index (Q_U) and the lower quality index (Q_L).

$$Q_U = \frac{96 - 94.46}{1.532} = 1.005 \qquad Q_L = \frac{94.46 - 92}{1.532} = 1.606$$

These values are then used to find percent within the upper specification limit (P_U) and the percent within the lower specification limit (P_L).

To find P_U , enter Table 71-1 at the column $n = 5$. Read down to find that 1.005 falls between 1.01 and 0.98. Therefore, an interpolation must be done to find where P_U falls between 83 and 84.

$$\frac{X - 83}{1.005 - 0.98} = \frac{84 - 83}{1.01 - 0.98}$$

Solve for X

$$X = 83.83$$

Therefore, $P_U = 83.83$

To find P_L , enter Table 71-1 at the column $n = 5$. Read down to find that 1.606 falls between 1.67 and 1.60. Therefore, an interpolation must be done to find where P_L falls between 98 and 99.

$$\frac{X - 98}{1.606 - 1.60} = \frac{99 - 98}{1.67 - 1.60}$$

Solve for X

$$X = 98.09$$

Therefore, $P_L = 98.09$

P_U and P_L are plugged into Equation 3.5 to find the Quality Level (Q_L).

$$Q_L = (83.83 + 98.09) - 100 = 81.92$$

$$Q_L = 81.9$$

To Solve for Pay Factor:

Equations used to calculate Pay Factor are in the Revision of Sections 105 and 106, for both Quality of Hot Mix Asphalt (HMA) and Voids Acceptance of Hot Mix Asphalt, of the Standard Specifications.

When P_n is from 3 to 9 or greater than 200

$$P_n = 5 \quad Q_L = 81.9$$

Go to Table 105-3

Go to formula for $P_n = 5$

$$0.25529 + 1.48268(Q_L/100) - 0.67759(Q_L/100)^2$$

$$0.25529 + 1.48268(81.9/100) - 0.67759(81.9/100)^2 = 1.015$$

Maximum PF for $P_n = 5$ is 1.030

Choose the smallest PF

$$PF = 1.015$$

When P_n is equal to or greater than 10 and less than 201

Example using Formula 1 when the number of tests equals 13:

$$P_n = 13 \quad Q_L = 81.9$$

Formula (1)

$$PF = \frac{(PF_1 + PF_2)}{2} + \left[\frac{(PF_2 + PF_3)}{2} - \frac{(PF_1 + PF_2)}{2} \right] \times \frac{(P_{n_2} - P_{n_x})}{(P_{n_2} - P_{n_3})}$$

Use Table 105-3 to solve for PF_1 , PF_2 and PF_3 of Formula 1. Use Table 105-3 to find P_{n_2} , P_{n_3} and P_{n_x} of Formula 1.

PF_1 = Formula for $P_n = 10$ to 11

$$0.15344 + 1.50104(81.9/100) - 0.58896(81.9/100)^2 = 0.988$$

Maximum PF for $P_n = 10$ to 11 is 1.045

Choose smallest PF

$$PF_1 = 0.988$$

PF₂ = Formula for Pn = 12 to 14

$$0.07278 + 1.64285(81.9/100) - 0.65033(81.9/100)^2 = 0.982$$

Maximum PF for Pn = 12 to 14 is 1.045

Choose smallest PF

$$PF_2 = 0.982$$

PF₃ = Formula for Pn = 15 to 18

$$0.07826 + 1.55649(81.9/100) - 0.56616(81.9/100)^2 = 0.973$$

Maximum PF for Pn = 15 to 18 is 1.050

Choose smallest PF

$$PF_3 = 0.973$$

Pn₂ = Lowest Pn in 12 to 14

$$Pn_2 = 12$$

Pn₃ = Lowest Pn in 15 to 18

$$Pn_3 = 15$$

$$Pn_x = 13$$

$$PF = \frac{(0.988 + 0.982)}{2} + \left[\frac{(0.982 + 0.973)}{2} - \frac{(0.988 + 0.982)}{2} \right] \times \frac{(12 - 13)}{(12 - 15)} = 0.9825$$

Maximum PF for Pn = 12 to 14 is 1.045

Choose smallest PF

$$PF = 0.9825$$

To Solve for Element Average Pay Factor:

At times, for instance when material is greater than 2V out, a separate process is started. This example will show how to determine an Average Pay Factor for an element that is represented by 3 different processes. Let's say the following Pay Factors were calculated:

PF₁ = 1.011 for 10,000 tons

PF₂ = 0.694 for 500 tons

PF₃ = 1.022 for 10,500 tons

$$\text{PFA} = \frac{[10,000(1.011) + 500(0.694) + 10,500(1.022)]}{10,000 + 500 + 10,500}$$

PFA = 1.009

To Solve for Composite Pay Factor:

When each of the Element Pay Factors of an item has been calculated, the Composite Pay Factor is then found using the W Factor of the Standard Specials. The Composite Pay Factor is the total pay factor for an item, such as HMA. For example, the weights for the item HMA are:

Gradation 20 %

Asphalt Content 30 %

Asphalt Density 50 %

Let's say each element was calculated to have the following Average Pay Factors:

Gradation 1.014

Asphalt Content 1.026

Asphalt Density 1.009

$$\text{PFC} = \frac{[.20(1.014) + .30(1.026) + .50(1.009)]}{.20 + .30 + .50}$$

PFC = 1.015

This Composite Pay Factor is then applied to calculate the incentive or disincentive. For example, if 21,000 tons of HMA were put down at a cost of \$30/ton:

$$(1.015 - 1)(21,000)(30) = \$9,450$$

The total incentive for this job would be \$9,450.

When Paying Binder Separately

Binder can be paid for separately from the rest of the HMA. To calculate an incentive or a disincentive, the total cost of the binder and HMA must be found. An example follows that uses the following values:

| | | |
|---------------|---------------------------|-----------|
| 403 | HMA (GR SX) (76) | \$55/Ton |
| 411 | Asphalt Cement (PG 58-40) | \$150/Ton |
| 1000 tons mix | 5.35% asphalt | |
| 1000 tons mix | 5.30% asphalt | |
| 1000 tons mix | 5.35% asphalt | |
| 1000 tons mix | 5.32% asphalt | |
| 720 tons mix | 5.00% asphalt | |

$$\frac{(1000)(5.35) + (1000)(5.30) + (1000)(5.35) + (1000)(5.32) + (720)(5.00)}{1000 + 1000 + 1000 + 1000 + 720} = 5.28\%$$

5.28% asphalt for 4720 tons of mix

Solve for total cost of AC:

$$(4720 \text{ tons mix})(.0528 \text{ percent asphalt}) = 249.22 \text{ tons AC}$$

$$(249.22 \text{ tons AC})(\$150/\text{ton}) = \$37,383.00$$

Now solve for the Price/Ton for the combination of HMA and binder.

$$\text{Unit Price HMA (\$/ton)} + \frac{\text{Total Cost of AC (\$)}}{\text{Total tons HMA}}$$

$$\$55/\text{ton} + \frac{\$37,383}{4,720 \text{ tons HMA}} = \$62.92/\text{ton}$$

\$62.92 is the cost per ton for the **entire** mix, both HMA and AC. This value is used to calculate the incentive or disincentive.