Grade Level Expectation: 5-8
Gears and Proportions
Content Area: Mathematics

Lesson time:
1 hour

Materials:
1. Gearing Up handout (at end of this lesson)

2. Assessment: Level A (at end of this lesson)

3. Assessment: Level B (at end of this lesson)

4. Answer Key (at end of this lesson)

Video resource:
Bianca gets in gear
http://www.teachersdomain.org/resource/vtl07.math.number.rat.

Objective: To teach students how bicycle gears work and the relationships between the sizes of front and rear gears, gear ratios and relative speeds of the bicycle wheels. Students later list gear combinations and ratios for bicycles of different speeds. Adapted from Teachers' Domain: Gears and Proportions.

5th Grade Standard:
1. Patterns, Functions, and Algebraic Structures
   3. Number patterns are based on operations and relationships

6th Grade Standard:
1. Number, Sense, Properties and Operations
   3. Quantities can be expressed and compared using ratios and rates

Part I: Learning Activity
1. Read the following to your students: "Have you ever ridden a bicycle with multiple gears? In this activity, you will consider how mathematics can help us understand how gears work. In a Cyberchase video segment, Bianca buys a fast new bicycle to keep up with her friend Kelly when they go riding in the park."

2. Ask if any of the students can explain what happens when they change gears during a bike ride. Ask them to discuss whether a geared bike is easier to ride than a bike with no gears.

3. Play the *Bianca Gets in Gear* QuickTime Video. Tell students to watch as Bianca visits the bike shop and to pay attention to the shop owner's explanation of gears.

4. Distribute the Gearing Up handout.

5. Ask the students to complete the handout. (Note: This activity will be enhanced if your students can examine a geared bicycle directly. We suggest that the class go out to the bike rack and look at a geared bike. If any of the students has a bicycle at school, perhaps a student could bring a bicycle into the classroom so students can examine the gears, gearshifters and derailleurs, and how they work.)

6. Discuss with students the relationships between the front and rear gears and how mathematics is involved in the motion of the bicycle gears.

Part II: Assessment
From a set of gear combinations and gear ratios, ask students to identify high and low gear ratio valuesa and to determine what it means to compare gear ratios. Alternatively aks students to list gear combinations for a 12-speed bike and to rank the gear ratios in terms of relative speeds per pedal turn.

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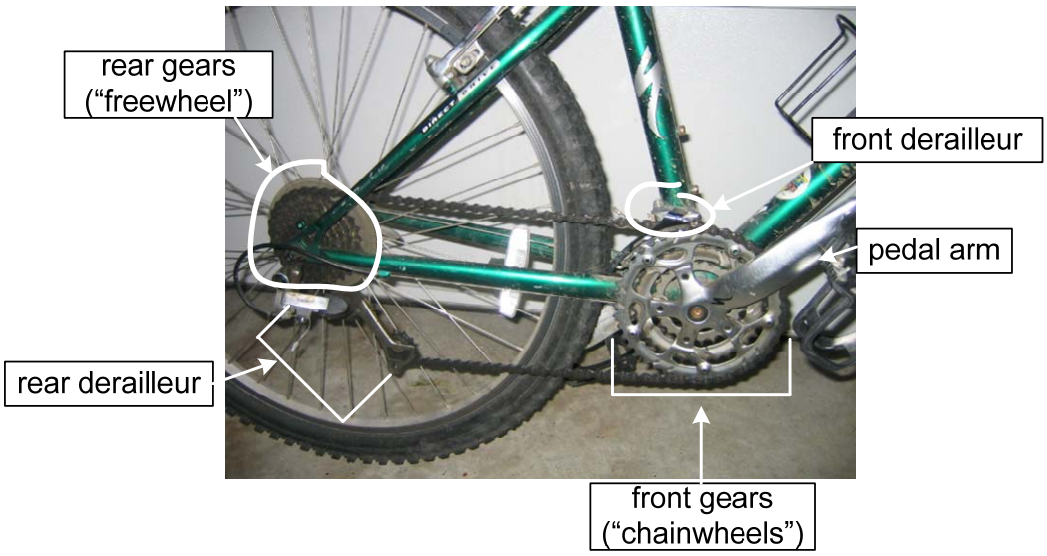
"Gearing Up" Handout

Bicycles have a bewildering assortment of gears on them. But what are all those gears for? Let's explore how the gears on a bike work.

Part 1

The pictures below show you some of the basic parts of a bicycle. There are two places on a bike where gears can be found. The front gears (also called the "chainwheels") are attached to the pedal arms (which hold the pedals out at their ends). Then there are gears attached to the rear wheel's hub. All the rear gears are contained in a cluster of gears on a "freewheel," which only turns the rear wheel when you pedal forward. It is "free" of the wheel if you pedal backwards. The two sets of gears are connected by the bicycle's chain. All the gear teeth are the same size. Why must this be the case?



The gear shifts on bikes can be on the handlebars, or mounted on the bicycle frame:

	
Left gear shift, for front gears. Note that on this bike, there are three marked positions (3 front gears)	Right gear shift, for rear gears Note that on this bike there are 7 marked positions (7 rear gears)

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The left shifter controls the front derailleur, for the front gears, or chainwheels. The right shifter controls the rear derailleur, for the rear gears, or freewheel.

The derailleurs, as their name suggest, "de-rail" the chain from one gear to the next, by pushing the chain to the left or the right.

From your own experience, and the pictures, answer the following questions.

1. Each time a pedal goes all the way around, which gearset--the front or the rear--goes around once?

2. When the rear wheel goes around once, which gear set--the front or the rear--goes around once?

3. If a bicycle has 2 gears in the front, and 5 in the rear, how many different combinations of gears are possible? Justify your answer.

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For each of the remaining problems, assume that the bicycle has 2 gears in the front, one of which has 30 teeth and the other 24. The bicycle also has 5 gears in the back, with 8, 12, 16, 20, and 24 teeth.

4. Now list all the pairs of possible combinations of gears in the following format: (# teeth on the front gear, # teeth on the rear gear). For instance, one combination would be (30,12), indicating the 30-tooth front gear connected by the chain to the 12-tooth rear gear. Enter the values in the table below. The first example is entered already.

Number of teeth in front gear	Number of teeth in rear gear	Gear combination
30	12	(30,12)

Now that we have all the combinations of gears on our bicycle, we will learn how to interpret how the ratio of the gears affects the action of the bicycle.

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5. If a front gear had 24 teeth, and a rear gear has 12 teeth:

    a. Each time the pedal goes around once, how many times does the rear gear (and wheel) go around?

    b. If the pedal goes around twice, how many times will the rear wheel go around?

    c. If the pedal goes around 4 times, how many times will the rear wheel go around?

We say that the ratio of the number of teeth in the front gear, to the number of teeth in the rear gear, is equal to 24:12. When we put this ratio in simplest terms, we call it the "gear ratio." Therefore, this 24:12 ratio represents a gear ratio of 2:1.

When the bike shop owner told Bianca about a speed gear, he was referring to gear ratios that cause the wheel to go around more times per pedal revolution. High performance bikes can have chainwheel-freewheel combinations that include gear ratios of up to 5 or more.

6. If the chain is on the 24 tooth front gear, and on the 8-tooth rear gear:

    a. What is the gear ratio?

    b. How many times does the rear wheel go around when the pedal goes around once?

    c. Will this combination result in a higher speed than the 2:1 gear ratio? Justify your answer.

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7. Complete the table below with the gear combinations for the bicycle that has two gears in the front, one of which has 30 teeth and the other 24. And five gears in the back have 8, 12, 16, 20, and 24 teeth.

# teeth in front gear	# teeth in rear gear	ratio of # front teeth: # rear teeth	(gear ratio)	# turns of wheel for each turn of pedal
24	8	24:8	3:1	3
24	12	24:12	2:1	2
24				
24				
24				
30				
30				
30				
30				
30				

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"Gears and Proportions"
Assessment Level B

A manufacturer has created a new trail bike with 2 front gears (56 and 42 teeth), and rear gears of 14, 21, 28, 35, 42, and 56 teeth.

1. How many gear combinations are possible for this new bicycle? Justify your answer.

2. Complete the first four columns of the table below for all the possible gear combinations.

# Teeth in Front Gear	# Teeth in Rear Gear	Ratio of # Front teeth to # Rear teeth	Gear Ratio	Speed Ranking (use 1 for lowest speed)

3. Identify gear combinations that have the same gear ratio.

4. Now, in the final column on the right, rank the gear combinations, with 1 being the **lowest** speed gear combination. The higher the rank number, the higher the speed. Gear ratios that tie should get the same rank.

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"Gears and Proportions"
Assessment Level A

The table below represents all the possible combinations of front and rear gears for the 10-speed bike from Handout 1.

# teeth in front gear	# teeth in rear gear	ratio of # front teeth: # rear teeth	(gear ratio)	# turns of wheel for each turn of pedal
24	24	24:24	1:1	1
24	20	24:20	6:5	1.2
24	16	24:16	3:2	1.5
24	12	24:12	2:1	2
24	8	24:8	3:1	3
30	24	30:24	5:4	1.2
30	20	30:20	3:2	1.5
30	16	30:16	15:8	1.875
30	12	30:12	5:2	2.5
30	8	30:8	15:4	3.75

1. What is the highest gear ratio in the table?

2. What is the lowest gear ratio in the table?

3. Identify two gear combinations that have the same gear ratio.

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"Gears and Proportions"
Assessment Level A, page 2

4. What does it mean for a gear ratio to be 1:1, in terms of the turns of pedal and turns of the wheels?

5. What does it mean for one gear ratio to be "higher" than another, in terms of the turns of the pedal and turns of the wheels?

6. What does it mean for one gear ratio to be "higher" than another, in terms of the effort you must expend or the speed you can go?

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"Gears and Proportions"
Answer Key

Handout 1: "Gearing Up"

1. The front gearset goes around once.

2. The rear gearset.

3. There are 10 gear combinations. The first gear in front can go with the 5 gears in the back for 5 plus the second gear in front can go with the 5 gears in the back for an additional five. Students may also use a tree diagram to justify their answer.

4.

Number of teeth in front gear	Number of teeth in rear gear	Gear combination
30	12	(30,12)
30	8	(30, 8)
30	16	(30, 16)
30	20	(30, 20)
30	24	(30, 24)
24	8	(24, 8)
24	12	(24, 12)
24	16	(24, 16)
24	20	(24, 20)
24	24	(24, 24)

5. (a) 2 times.

   (b) 4 times

   (c) 8 times

6. (a) 24:8 = 3:1

   (b) 3 times

(c) It is more of a speed ratio because on a 2:1 gear ratio the rear wheel only goes around twice when the pedal goes around once.

7.

# teeth in front gear	# teeth in rear gear	ratio of # front teeth: # rear teeth	(gear ratio)	# turns of wheel for each turn of pedal
24	8	24:8	3:1	3
24	12	24:12	2:1	2
24	16	24:16	3:2	1.5 (3/2)
24	20	24:20	6:5	1.2 (6/5)
24	24	24:24	1:1	1
30	8	30:8	15:4	3.75 (15/4)
30	12	30:12	5:2	2.5 (5/2)
30	16	30:16	15:8	1.875 (15/8)
30	20	30:20	3:2	1.5 (3/2)
30	24	30:24	5:4	1.25 (5/4)

Assessment Level A

1. 15:4

2. 1.1

3. 6:5 ratio exists when there are 24 front teeth and 20 rear teeth and also when there are 30 front teeth and 24 rear teeth.

   And 3:2, with 24:16 and 30:20 tooth combinations

4. Every time the pedal goes around once, the rear wheel goes around once.

5. The rear wheel turns more times per pedal stroke when the gear ratio is higher.

6. You must expend more effort when the gear ratio is higher but you travel much faster because the rear wheel is turning more times per pedal stroke. You would use a higher gear ratio on flat land, or going downhill where you want to race and where the pedaling is easier. You would not want to use a higher gear ratio when biking uphill because that would expend too much energy.

Assessment Level B

1. There are 12 gear combinations. The first front gear can match with the 6 rear gears for 6 combinations and the second front gear can match with the 6 rear gears for another 6 combinations for a total of 12 combinations. Students may also use a tree diagram to justify their answer.
2.

# teeth in front gear	# teeth in rear gear	Ratio of Front teeth to Rear teeth	Gear Ratio	Speed rank (1 is lowest)
56	14	56:14	4:1	10
56	21	56:21	8:3	8
56	28	56:28	2:1	7 (tie)
56	35	56:35	8:5	6
56	42	56:42	4:3	4
56	56	56:56	1:1	2 (tie)
42	14	42:14	3:1	9
42	21	42:21	2:1	7 (tie)
42	28	42:28	3:2	5
42	35	42:35	6:5	3
42	42	42:42	1:1	2 (tie)
42	56	42:56	3:4	1

3. 56:28 and 42:21 (2:1)
   56:56 and 42:42 (1:1)

4. See table above. Remember that the ratios are ranked from lowest to highest, (1 to 10, with two ties).