Appendix D – Basic Roadway Geometry Information

USE OF CARTESIAN SYSTEMS

The following summarizes some of the basic formulas for Cartesian coordinate systems.

For implicitly distinct points:
P(1) represented by coordinates (X_1, Y_1)
P(2) represented by coordinates (X_2, Y_2)
P(3) represented by coordinates (X_3, Y_3) etc.

P(1), P(2) and P(3) lie on the same line (are colinear) if

$$Y_3 = Y_2 + \frac{(X_3 - X_2)(Y_1 - Y_2)}{(X_1 - X_2)} \quad \text{OR} \quad X_3 = X_2 + \frac{(Y_3 - Y_2)(X_1 - X_2)}{(Y_1 - Y_2)}$$

$$\text{OR det} \begin{vmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{vmatrix} = 0$$

Distance from P(1) to P(2) (in the horizontal plane)

$$\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

The Euclidean norm (including difference in elevation)

$$\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (\text{Elev.}_2 - \text{Elev.}_1)^2}$$

Line through P(1) and P(2) is parallel to line through P(3) and P(4) if

$$(X_1 - X_2)(Y_3 - Y_4) = (X_3 - X_4)(Y_1 - Y_2)$$

Line through P(1) and P(2) is perpendicular to line through P(3) and P(4) if:

$$(X_1 - X_2)(X_3 - X_4) = (Y_1 - Y_2)(Y_4 - Y_3)$$

Area of triangle with vertices P(1), P(2) and P(3)

$$\left|(X_1 Y_2 + X_2 Y_3 + X_3 Y_1 - X_1 Y_3 - X_2 Y_1 - X_3 Y_2)\right| / 2$$

$$\left|((X_1 - X_2)(Y_3 - Y_2) - (X_3 - X_2)(Y_1 - Y_2))\right| / 2$$

Area of quadrilateral with sequential vertices P(1), P(2), P(3) and P(4)

$$\left|((X_1 Y_2 + X_2 Y_3 + X_3 Y_4 + X_4 Y_1 - X_1 Y_4 - X_2 Y_1 - X_3 Y_2 - X_4 Y_1))\right| / 2$$

Distance of P(3) from the line through P(1) and P(2) is equal to twice the area of triangle P(1), P(2), P(3) divided by distance from P(1) to P(2)

$$\frac{\left|((X_1 - X_2)(Y_3 - Y_2) - (X_3 - X_2)(Y_1 - Y_2))\right|}{\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}}$$

Transit at P(0), the angle turned from the line (parallel to Y-axis) to sight P(1) is given by

$$\theta = \text{Arctan} \frac{X_1 - X_0}{Y_1 - Y_0}$$

The angle turned from sight on P(1) to sight P(2) is given by

$$\theta = \text{Arctan} \frac{(Y_1 - Y_0)(X_2 - X_0) - (X_1 - X_0)(Y_2 - Y_0)}{(X_1 - X_0)(X_2 - X_0) + (Y_1 - Y_0)(Y_2 - Y_0)}$$

If $\tan(\theta)$ is > 0, θ may be either to the right $0° < \theta < 90°$ or to the left $-180° < \theta < -90°$.

If $\tan(\theta)$ is < 0, θ may be either to the left $-90° < \theta < 0°$ or the right $90° < \theta < 180°$.

PI Point of intersection of tangents
TS Tangent to spiral point
SC Spiral to circle point
CS Circle to spiral point
ST Spiral to tangent point

Δ = Total deflection angle of curve
θs = Deflection angle of spiral
Ts = Distance from ST to PI
Es = External distance from PI to center of circular portion of curve
Ls = Length of spiral
Lt = Distance from ST or TS to PI of spiral
St = Distance from PI of spiral to SC or CS
Dc = Degree of curve of circular portion
Tc = Distance from S.C. or C.S. to P.I of circular portion
Lc = Arc length of circular portion S.C. to C.S.
Rc = Radius of circular Portion

$D = \frac{L}{L_s} \times D_c$; Relationship between Dc and the curvature of the spiral

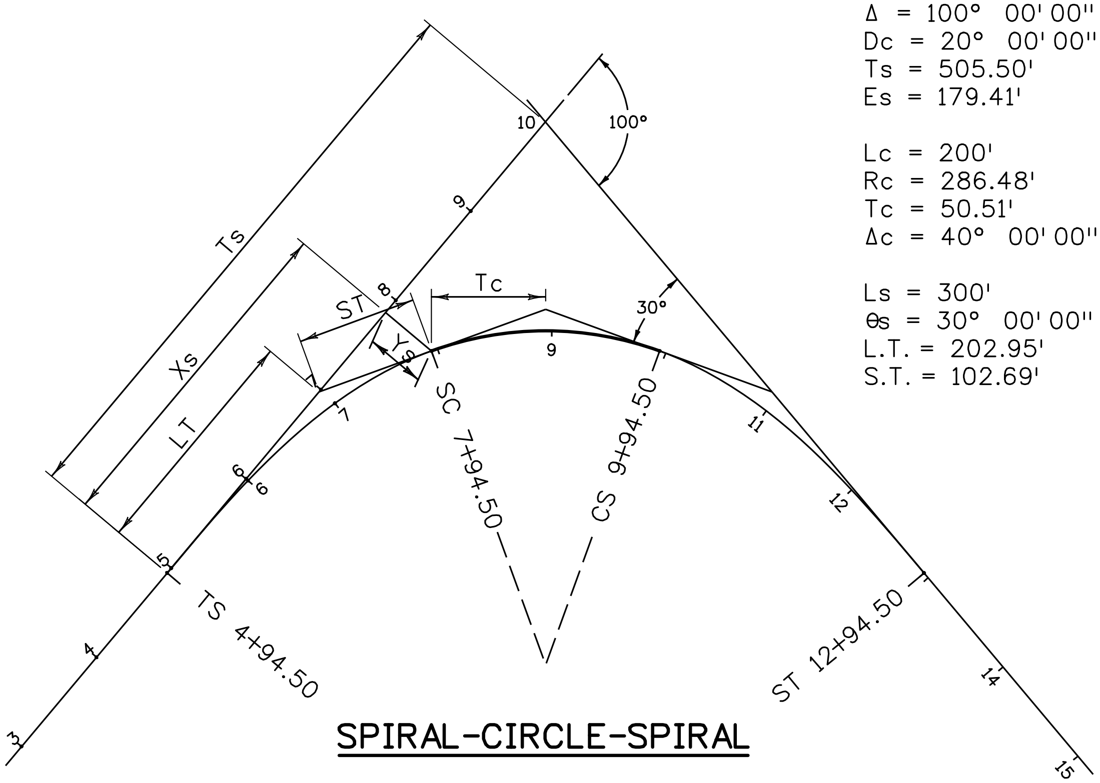
$\theta_s = \frac{L_s}{200} \times D_c$; Relationship between θs, Ls, and Dc

$\theta = \frac{L^2}{L_s^2} \times \theta_s$; Angle at any length (L) along spiral with respect to Ls and θ

Δ = 100° 00' 00''
Dc = 20° 00' 00''
Ts = 505.50'
Es = 179.41'

Lc = 200'
Rc = 286.48'
Tc = 50.51'
Δc = 40° 00' 00''

Ls = 300'
θs = 30° 00' 00''
L.T. = 202.95'
S.T. = 102.69'



SPIRAL-CIRCLE-SPIRAL

TS Point of change from tangent to spiral
ST Point of change from spiral to tangent
SC Point of change from spiral to circle
CS Point of change from circle to spiral
l Spiral arc length from TS to any point
l_s total length of spiral from TS to SC
θ Central angle of spiral arc l
θ_s Central angle of spiral arc ls, called "spiral angle"
D Degree of curve of the spiral at any point
R Radius of curve of the spiral at any point
D_c Degree of of curve of the shifted circle to which the spiral becomes tangent at the SC
R_c The radius of the circle
L.T. Long tangent distance of spiral only
S.T. Short tangent distance of spiral only
p Offset distance from the tangent of P.C. of circular curve produced
k Distance from T.S.to point on tangent opposite the P.C. of the circular curve produced
x,y Coordinates at any point on the spiral
x_s,y_s Coordinates at the S.C. or C.S.

$D = \frac{L}{L_s} X\ D_c$; Relationship between Dc and the curvature of the spiral

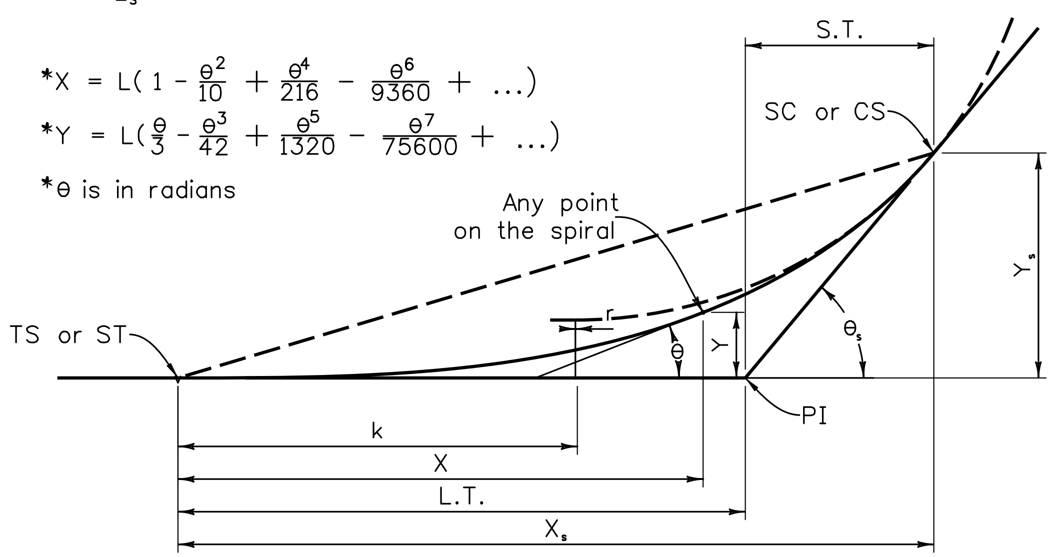
$\theta_s = \frac{L_s}{200} X\ D_c$; Relationship between θs, Ls, and Dc

$\theta = \frac{L^2}{L_s^2} X\ \theta_s$; Angle at any length (L) along spiral with respect to Ls and θ

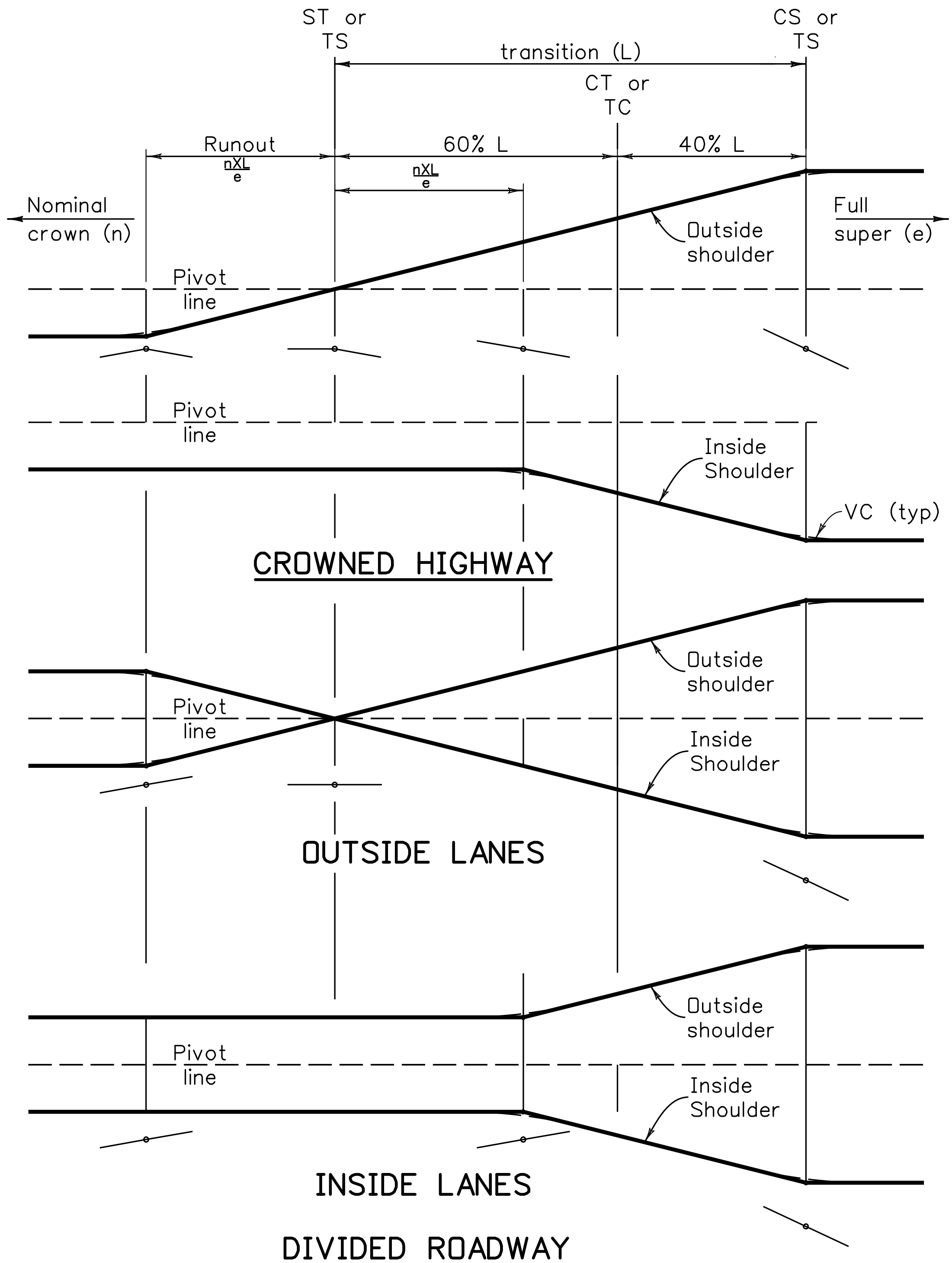
$$*X = L\left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9360} + \ldots\right)$$

$$*Y = L\left(\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \frac{\theta^7}{75600} + \ldots\right)$$

*θ is in radians



SPIRAL EXAMPLE



SUPERELEVATION DIAGRAMS